Workshop on

Set-valued Numerical Analysis and Robust Optimal Control

Hausdorff Research Institute for Mathematics

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53115 Bonn

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Organizing committee:

Robert Baier
Eva Crück
Tzanko D. Donchev
Matthias Gerdts
Thomas Lorenz
Janosch Rieger

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Thursday, March 27

Assen L. Dontchev

Implicit function theorems: Old and new

In the classical setting of the implicit function theorem, an equation $f(p, x) = 0$ is solved for x in terms of a parameter p. The questions center on the extent to which this solution mapping can be expressed, at least locally, by a function from p to x, and if so, what properties can be guaranteed for that function. In this talk we move into wider territory where, although the questions are basically the same, it is no longer an equation $f(p, x) = 0$ that is being solved, but a condition capturing a more complicated dependence of x on p, and the solution mapping may be set-valued. Motivations come from optimization and models of equilibrium. We will show that the implicit function paradigm can be carried further in the framework of a mapping from the parameter to sequence of iterates generated by Newton's method applied to generalized equations.

Division of Mathematical Sciences, National Science Foundation
Arlington, VA 22230, U.S.A.
Email: adontche@nsf.gov

Fritz Colonius

Some numerical problems for stability of deterministic and stochastic systems

In this talk I will survey several problems concerning set-valued numerics for deterministic systems with uncertainties and for random systems. In particular, computation of reachable sets for control systems plays a decisive role.

Institut für Mathematik, Universität Augsburg
86135 Augsburg, Germany
Email: fritz.colonius@math.uni-augsburg.de

Peter E. Kloeden

The numerical solutions of stochastic differential equations with a discontinuous drift coefficient

Joint work with Nikolaos Halidias, University of the Aegean,Samos, Greece.

In [1] we proved the existence of strong solutions for d-dimensional autonomous Itô stochastic differential equations

$$dX_t = f(X_t)\,dt + g(X_t)\,dW_t, \qquad t \in [0, T] \tag{1}$$

for which the drift coefficient is a monotone increasing function (but not necessarily continuous) and the diffusion coefficient is Lipschitz continuous. By an increasing function we mean that $f(x) \le f(y)$ whenever $x \le y$, where the inequalities are interpreted componentwise. A motivating example is scalar SDE

$$dX_t = H(X_t)\,dt + dW_t, \tag{2}$$

where $H : \mathbb{R} \to \mathbb{R}$ is the Heaviside function, which is defined by

$$H(x) := \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

Such equations arise, for example, when one considers the effects of background noise on switching systems or other discontinuous ordinary differential equations.

Here we show that the Euler-Maruyama scheme applied stochastic differential equations such as (2) can be used to obtain numerical approximations which converge strongly (i.e. mean-square sense) to a solution with the same initial value. To be specific, we consider the numerical approximation of such stochastic differential equations with additive noise, i.e. of the form

$$dX_t = f(X_t)\,dt + A\,dW_t, \qquad t \in [0, T] \tag{3}$$

for which the drift coefficient is a monotone increasing function (but not necessarily continuous) and A is a $d \times k$ matrix and W_t a d dimensional Wiener process.

Both the existence proof and the numerical results make extensive use of upper and lower solutions.

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Institut für Mathematik, Johann Wolfgang Goethe–Universität
60054 Frankfurt am Main, Germany
Email: kloeden@math.uni-frankfurt.de

Patrick Saint-Pierre

How to control complex dynamical system: A generation of viability algorithm

What are we driving at when aiming at controlling a dynamical system ? Exploiting various numerical technics for finding a solution to a differential equation lead to develop two main concepts of control that the control community name: *open loop* and *closed loop* or *feed-back* control. Controlling means managing the evolutions ruled by some dynamical equations so as to satisfy some a priori given *specifications*. Then two wide class of methods appeared. One uses the open loop control for exploring the space thanks to, for instance, Monte-Carlo or other statistical methods allowing to measure how these specifications can be fulfilled. The second tries to build feed-back controls thanks to mathematical properties of systems such as Kalman filtering or PID controlling.

Originally, Viability Theory appeared not for a control purpose but for answering to more qualitative questions: Is it possible, from an initial position, to maintain forever the evolution in a given constraint set, and how ? It appeared since the last two decades that Viability Theory can be intimately connected to Control Theory and bring up new theoretical and numerical issues. Our aim is to survey the more recent Viability Theory developments for controlling complex dynamical systems, examining what numerical algorithms can be designed for finding appropriate regulations which respect given specifications and give examples of applications in Engineering, in Environment and in Finance.

Laboratoire d'Applications des Systèmes Tychastiques Régulés (LASTRE)
Centre de Recherche Stratégies et Dynamiques Financières: Viabilité, Jeux, Contrôle
Université Paris-Dauphine
75775 Paris, France
Email: patrick.saint.pierre@gmail.com

Tzanko D. Donchev

Invariance and some other properties of impulsive systems

We study weak invariance of differential inclusions with non-fixed time impulses under compactness type assumptions in general Banach spaces. The tangential condition is the classical Bony condition using the Bouligand contingent cone.
If the right-hand side is one-sided Lipschitz an invariance result and an extension of the well known relaxation theorem are proved.
In case of $\mathbb{R}^n$ also necessary and sufficient condition for invariance of upper semi continuous systems with the help of the proximal normal cone are obtained.
Some properties of the solution set of impulsive system (R_δ) without constraints in appropriate topology are presented.

Department of Mathematics, University of Architecture and Civil Engineering,
1046 Sofia, Bulgaria
Email: tzankodd@gmail.com

Tatiana F. Filippova

State estimation approaches for control systems under uncertainty

The topics of the talk come from the theory of dynamical systems with unknown but bounded uncertainties related to the case of set-membership description of uncertainty. The talk presents recent results in the theory of tubes of solutions ("trajectory tubes") to differential control systems with uncertain parameters or functions.

The problems of evolution modeling for uncertain dynamic systems with system states being compact subsets of the state space are well known and remain important both for the control theory and for numerous applications. Applying results related to discrete-time versions of the funnel equations describing the behavior of set-valued system states and techniques of ellipsoidal estimation theory developed for linear control systems we present new approaches that allow to find external and internal set-valued estimates for trajectory tubes in several new classes of uncertain control system.

In particular we present the modified state estimation approaches for nonlinear dynamical control system with unknown but bounded initial state and with quadratic nonlinearity in dynamics with respect to state variable. Basing on the well-known results of ellipsoidal calculus developed for linear uncertain systems we present the modified state estimation approaches which use the special structure of the dynamical system.

Another interesting class of the considered problems constitutes uncertain differential systems with impulsive controls. We study the state estimation problems for such systems under a special restriction on impulsive control functions defined by a given generalized "ellipsoid" in the space of functions of bounded variations. In particular, under such restriction vectors of impulsive jumps of admissible controls have to belong to a given finite-dimensional ellipsoid.

We introduce state estimation algorithms based on the properties and on the special structure of solutions of differential systems with impulsive controls, in particular we construct the ellipsoidal estimates for a convex hull of the union of related ellipsoids of a finite dimensional space.

Numerical simulation results illustrating the theoretical approaches are also included.

Institute of Mathematics and Mechanics, Russian Academy of Sciences
Ekaterinburg 620219, Russia
Email: ftf@imm.uran.ru

Matthias Gerdts

Approximation of Reachable Sets using Optimal Control Algorithms

Numerical experiences with a method for the approximation of reachable sets of nonlinear control systems are reported. The method is based on the formulation of suitable optimal control problems with varying objective function, whose discretization by Euler's method lead to finite dimensional non-convex nonlinear programs. These are solved by a sequential quadratic programming method. An efficient adjoint method for gradient computation is used to reduce the computational costs.

An alternative algorithm based on a non-smooth Newton method is suggested. This algorithm shows a mesh independence and may further reduce the computational costs.

The method is illustrated for two test examples. Both examples have non-linear dynamics, the first one has a convex reachable set, whereas the second one has a non-convex reachable set.

Key words: optimal control, reachable set, direct discretization, nonsmooth Newton method

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School of Mathematics, The University of Birmingham
Edgbaston, Birmingham B15 2TT, Great Britain
Email: gerdtsm@maths.bham.ac.uk

Friday, March 28

Christof Büskens

Adaptive Closed-Loop Optimal Control

Open-loop solution of a optimal control problems are just the first step to cope with the practical realization of real life applications. Closed-loop (feedback) controllers, like the classical Linear Quadratic Regulator (LQR), are needed to compensate perturbations appearing in reality. Although these controllers have proved to be a powerful tool in many applications and to be robust enough to countervail most differences between simulation and practice, they are not optimal if disturbances in the system data occur. If these controllers are applied in a real process, the possibility of data disturbances force recomputing the feedback control law in real-time to preserve stability and optimality, at least approximately. For this purpose, a numerical method based on the parametric sensitivity analysis of nonlinear optimization problems is suggested to calculate higher order approximations of the feedback control law in real-time. Using this method, the optimal controller can be adapted within a few nanoseconds on an typical personal computer. The method is illustrated by the adaptive optimal control of a warehouse crane and the classical inverted pendulum.

Key words: optimal control, closed-loop, perturbations, Riccati

Center of Industrial Mathematics, Optimization and Optimal Control, University of Bremen
Bibliothekstraße 1, 28334 Bremen, Germany
Email: bueskens@math.uni-bremen.de

Helmut Maurer

Sensitivity analysis and real-time control of bang-bang and singular control problems

Optimal control problems with control appearing linearly are considered. The evaluation of Pontryagin's minimum principle shows that the optimal control is composed by bang–bang and singular arcs. The control problem induces a finite-dimensional optimization problem with respect to the switching times between bang-bang and singular arcs [1,3–6].
We discuss the arc-parametrization method [4] for efficiently solving the induced optimization problems and checking second-order sufficient conditions (SSC); cf. [3]. SSC for bang-bang control problems are then obtained from the SSC for the induced optimization problem and an additional property of the switching function [1,5]. SSC for singular control problems require stronger conditions which are currently under investigation.
The verification of SSC is the basis for the sensitivity analysis of perturbed control problems. The computations of parametric sensitivity derivatives of switching times then allows to develop real-time control techniques that are considerable easier to implement than the classical neighboring extremal approach. Several examples are discussed:
(1) optimal control of a Van-der-Pol oscillator,
(2) optimal control of a semiconductor laser,
(3) optimal chemotherapy of HIV.

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Institut für Numerische und Angewandte Mathematik
Westfälische Wilhelms-Universität Münster
48149 Münster, Germany
Email: maurer@math.uni-muenster.de

Lars Grüne

A set-oriented min-max approach to robust optimal control under coarse discretization and quantization

Joint work with O. Junge (TU München), M. von Lossow and F. Müller (University of Bayreuth).

In this talk we consider the problem of controlling a nonlinear discrete time control system

$$x(n+1) = f(x(n), u(n)), \quad x(n) \in X \subset \mathbb{R}^d,\ u(n) \in U, \quad x(0) = x_0$$

to a specified target set $\mathcal{T} \subset X$ while minimizing the performance index

$$J(x_0, u) = \sum_{n=0}^{N(\mathcal{T},u)} g(x(n), u(n))$$

where $N(\mathcal{T}, u) = \inf\{n \geq 0 \,|\, x(n) \in \mathcal{T}\}$ is the first hitting time of the target set $\mathcal{T}$. Specifically, we are looking for an approximately optimal control in feedback form, i.e., $u(n) = F(x(n))$ for some feedback map $F : X \to U$.

Junge and Osinga [2] proposed a global numerical method for such problems which relies on a set-oriented discretization of the continuous-state problem resulting in a graph model with finitely many states on which the control problem can be efficiently solved by Dijkstra's Algorithm for finding shortest paths in graphs. The drawback of this method is that in general a rather fine discretization is needed in order to obtain a valid feedback law F for the original problem.

In order to overcome this problem, in this talk we propose a variant of the approach which explicitly takes the discretization into account in the construction of the graph and thus allows for the design of a feedback law which is robust against the discretization errors.

We explain this variant of the method in detail and show how a recently developed min-max version of Dijkstra's Algorithm [1, 3] can be used for efficiently solving the resulting min-max shortest path problems on hypergraphs. Furthermore, we present several recent modifications of the method. In particular, we discuss a re-interpretation of the method in order to handle coarse quantization errors, an application to event based control and an extension to the computation of feedback laws depending on several consecutive states $x(n-m), \ldots, x(n)$ of the system's trajectory.

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Mathematisches Institut, Universität Bayreuth
95440 Bayreuth, Germany
Email: lars.gruene@uni-bayreuth.de

Moritz Diehl

Practical approaches to robust optimal control and real-world experiments in engineering applications

Joint work with Boris Houska and Peter Kuehl.

The talk will address the question how challenging control problems can be solved by help of nonlinear dynamic optimization which takes uncertainty explicitly into account. The practical approach we propose is fully open-loop and based on an approximation of the reachable set based on system linearization. Idea is to simultaneously optimize a nominal trajectory along with a tubeöf uncertainty ellipsoids around it, and to make sure critical constraints are satisfied for all possible uncertainty realizations. The numerical solution is based on Bock's direct multiple shooting method, and we discuss the additional numerical issues related to the inclusion of the uncertainty ellipsoids.

We will present two challenging applications:

a) Nominal and robust open-loop control of an exothermic chemical reactor that shall avoid runaways (including experiments)

b) Periodic optimal control of kites for a novel way of large scale wind power generation, and generation of robust and open-loop stable orbits (simulations)

Optimization in Engineering Center (OPTEC), Katholieke Universiteit Leuven
3001 Leuven-Heverlee, Belgium
Email: moritz.diehl@esat.kuleuven.be

Jan-Joachim Rückmann

On stable feasible sets in generalized semi-infinite programming

Joint work with Harald Günzel and Hubertus Th. Jongen (RWTH Aachen).

We consider the feasible set of a generalized semi-infinite programming problem with a one-dimensional index set of inequality constraints depending on the state variable. The latter dependence on the state variable gives rise to a complicated structure of the feasible set. Under appropriate transversality conditions we present the local description of feasible sets in new coordinates by means of finitely many basic functions.

Key words: Generalized semi-infinite programming, feasible set, transversal zero-point, coordinate transformation, decomposition theorem.

School of Mathematics, The University of Birmingham
Edgbaston, Birmingham B152TT, United Kingdom
Email: ruckmanj@maths.bham.ac.uk

José Alberto Murillo Hernández

The heat equation in noncylindrical domains governed by morphological equations

Modeling of several phenomena (evolution of tumor cells or environmental models, among others) involve a partial differential equation in a noncylindrical or time-varying domain, that is, the domain where the equation is defined change along time. In the works devoted to this topic (see [3], [5], [8] and the references therein), the noncylindrical domain is usually described either by means of a time-dependent family of diffeomorphisms or as the reachable set of a nonautonomous vector field, roughly speaking, its evolution is assumed to be given a priori. In this talk, by using ideas from Mutational Analysis introduced by J.-P. Aubin, we consider a more general framework, where the heat equation is defined in a family of domains that determine (properly their closures) their own dynamics, governed by a mutational shape equation. This allows to consider models where the velocity of change (in the mutational sense) can be influenced by the (global) shape of domains or, equivalently, there exists a feedback law ruling the evolution of the domains along time. It must be noted that this work is a first step towards the more general case of time-dependent domains described by multivalued flows or mutational morphological equations in the Aubin's notation.

Key words: Heat equation, noncylindrical domains, morphological equations.

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Departamento de Matemática Aplicada y Estadística
Universidad Politécnica de Cartagena
30202 Cartagena, Spain
Email: alberto.murillo@upct.es

Thomas Lorenz

Control problems for nonlocal set evolutions with state constraints

In this short presentation, we extend fundamental notions of control theory to evolving compact subsets of the Euclidean space.
Dispensing with any restriction of regularity, shapes can be interpreted as nonempty compact subsets of the Euclidean space $\mathbb{R}^N$. Their family $\mathcal{K}(\mathbb{R}^N)$, however, does not have any obvious linear structure, but in combination with the popular Pompeiu-Hausdorff distance $\mathit{d\!l}$, it is a metric space. Here Aubin's framework of morphological equations is used for extending ordinary differential equations beyond vector spaces, namely to the metric space $(\mathcal{K}(\mathbb{R}^N), \mathit{d\!l})$.
Now various control problems are formulated for compact sets depending on time: open-loop and closed-loop control problems – each of them with state constraints. Using the close relation to morphological inclusions with state constraints, we specify sufficient conditions for the existence of compact-valued solutions.

Interdisziplinäres Zentrum für Wissenschaftliches Rechnen, Universität Heidelberg
69120 Heidelberg, Germany
Email: thomas.lorenz@iwr.uni-heidelberg.de

Saturday, March 29

Frank Lempio

Discrete approximation of differential inclusions

In the first part of the talk, differential inclusions are introduced as a general framework for control problems with state constraints. Moreover, specific notions from abstract discretization theory, e.g. consistency, stability and discrete convergence, are adapted to the set-valued situation.

In the second part, set-valued Euler's method is described as a numerical tool for the approximation of the whole solution set of certain classes of differential inclusions with state constraints. Discrete convergence is measured by uniform discrete Hausdorff-distance and is interpreted as the result of appropriate stability and consistency properties.

In the third part of this talk, introducing an additional optimality criterion, so-called direct methods for optimal control problems are analyzed conceptually. Discrete convergence in value, i.e. convergence of the sequence of discrete minimal values, is a simple consequence of discrete convergence of the feasible sets. Discrete convergence of the corresponding discrete optimal trajectories is a much more elaborate subject.

As it is known from the numerical analysis of so-called indirect methods which exploit first-order necessary optimality conditions directly, sufficient optimality conditions imply local discrete convergence of discrete optimal trajectories and controls at the expense of additional smoothness and regularity assumptions for the continuous optimal trajectory and control.

In an appropriate sense, sufficient optimality conditions can be interpreted as local stability property of the minimal value function corresponding to the direct discretization method. An analogous stability property, globally along the sequence of discrete optimal trajectories, yields discrete convergence and error estimates for the sequence of discrete trajectories, though with respect to weaker norms than discrete supremum norm, but without any additional assumptions on the optimal control.

Mathematisches Institut, Universität Bayreuth
95440 Bayreuth, Germany
Email: frank.lempio@uni-bayreuth.de

Vladimir M. Veliov

Some problems and new results concerning discrete approximations of control systems

We address several problems for approximation of continuous-time control systems by discrete-time systems. The approximation involves a mapping that *projects* the set of admissible controls on a finite-dimensional one. We distinguish three different types of such mappings depending on the information pattern: local, non-anticipative, and anticipative. It turns out that the accuracy of approximation may depend on the information pattern.
The relation between the information pattern and the occurrence of the effect of non-accumulation of errors established earlier by the author is discussed. The ideas will be presented in the case of a bilinear control system having a rather complicated behaviour.

Institut für Wirtschaftsmathematik, TU Wien
Operations Research & Nichtlineare Dynamische Systeme
1040 Wien, Austria
Email: vveliov@server.eos.tuwien.ac.at

Elza Farkhi

Discrete approximations and relaxation of one-sided Lipschitz differential inclusions

In the theory of differential inclusions, the one-sided Lipschitz (OSL) condition was introduced in 1990, by Kastner-Maresch and Lempio and in a weaker version, by T.Donchev. The latter condition extends both the classical Lipschitz condition and known monotonicity notions for single-valued and set-valued functions.

The OSL condition, as in the Lipschitz case, yields Lipschitz approximation of the trajectory set with respect to perturbations in the initial point (and/or the right-hand side). It, however, does not provide neither the approximation of the velocity set, nor the relaxation stability in optimization problems.

To achieve these goals, the stronger modified one-sided Lipschitz (MOSL) condition may be invoked, yet still weaker than the Lipschitz condition. In a joint work with T. Donchev and B. Mordukhovich, we establish sufficient conditions for the strong approximation (in the $W^{1,p}$-norm, $p \geq 1$) of feasible trajectories for the differential inclusions in Hilbert spaces by those for their discrete approximations. For dynamic optimization problems we derive a new extension of the Bogolyubov-type relaxation/density theorem from the Lipschitz case to the case of differential inclusions satisfying the MOSL condition.

Finally, some open problems will be discussed.

School of Mathematical Sciences, Tel-Aviv University
Tel-Aviv 69978, Israel
Email: elza@post.tau.ac.il

Janosch Rieger

Shadowing in set-valued dynamical systems

In this talk the shadowing property will be examined in the context of set-valued dynamics given by

$$x_{k+1} \in F(x_k), \quad n \in \mathbb{Z} \tag{1}$$

and

$$\dot{x}(t) \in F(x(t)) \text{ a. e. in } \mathbb{R}. \tag{2}$$

System (1) has the shadowing property, if for some $\epsilon > 0$ and $d > 0$ and every d-pseudotrajectory $(y_k)_{k\in\mathbb{Z}}$ of (1), i.e. any sequence (y_k) with

$$\operatorname{dist}(y_{k+1}, F(y_k)) < d, \quad k \in \mathbb{Z},$$

there exists a real trajectory of (1) such that $\|x - y\| \leq \epsilon$. Any system which has the shadowing property is robust with respect to small errors on an infinite time interval.

It will be shown that classical techniques can easily be adapted to the set-valued case for contractive right-hand sides. A notion of hyperbolicity for set-valued mappings will be proposed and discussed.

AG Numerische Analysis Dynamischer Systeme, Universität Bielefeld
33501 Bielefeld, Germany
Email: jrieger@math.uni-bielefeld.de

Robert Baier

Set-Valued Hermite Interpolation, a Case Study of Directed Sets

Joint work with Gilbert Perria, Oristano, Italy.

The problem of interpolating a set-valued map $F\colon I \rightrightarrows \mathbb{R}^n$ with convex, compact images by polynomial maps is addressed (cf. [2, 3]). In this approach, the convex compact sets $\mathcal{C}(\mathbb{R}^n)$ are embedded into the Banach space of directed sets $\overrightarrow{\mathcal{D}}^n$ (cf. [1]), since negative weights in the interpolation polynomial could appear for degrees higher than 1 and $\mathcal{C}(\mathbb{R}^n)$ does not form a vector space with the usual operations (Minkowski sum, scalar multiplication). Directed sets provide generalizations of the above mentioned set operations and a visualization of (in general, non-convex) subsets of $\mathbb{R}^n$. They are visualized by at most two of three possible parts (positive/negative/mixed-type part).

A directed set is parameterized by unit directions in $\mathbb{R}^n$ and consists of pairs with two components, a lower dimensional directed set in $\mathbb{R}^{n-1}$ and a scalar value in $\mathbb{R}$. The embedded convex set consists of the embedded supporting faces and the values of the support function in all normed directions.

The interpolation polynomial $\overrightarrow{H}$ in $\overrightarrow{\mathcal{D}}^n$ fulfills

$$D^i \overrightarrow{H}(\theta_k) = D^i \overrightarrow{F}(\theta_k) \quad (i = 0, \ldots, \mu_k - 1,\ k = 0, \ldots, m)$$

with multiplicities μ_k on the $m + 1$ different interpolation nodes θ_k. Hereby, the resulting interpolation polynomial (e.g., in Newton form) acts separately on the components of the embedded convex-valued function $\overrightarrow{F}\colon I \to \overrightarrow{\mathcal{D}}^n$, since arithmetic operations, divided differences and derivatives (i.e., limit of difference quotients) in $\overrightarrow{\mathcal{D}}^n$ operate in the same way on both components of the images of $\overrightarrow{F}$. Under suitable smoothness conditions, the same results on the order of convergence follow for the interpolation as in the real-valued case. E.g., estimations for the derivatives by the interpolation polynomial as well as for piecewise Hermite interpolation can be transferred to the set-valued case.

Connections to other interpolation results, e.g., for Banach spaces in [2, 4] and for piecewise linear interpolation, are shown as well as some numerical test examples which include an interpolation of reachable sets of linear differential inclusions.

Key words: set-valued Hermite interpolation, directed sets, difference and embeddings of convex compacts, derivatives of set-valued maps

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Mathematisches Institut, Universität Bayreuth
95440 Bayreuth, Germany
Email: robert.baier@uni-bayreuth.de

List of participants

Robert Baier

Universität Bayreuth
Mathematisches Institut
Lehrstuhl für Angewandte Mathematik
95440 Bayreuth
robert.baier@uni-bayreuth.de

Wolf-Jürgen Beyn

Universität Bielefeld
Fakultät für Mathematik
Postfach 100131
33501 Bielefeld
beyn@mathematik.uni-bielefeld.de

Christof Büskens

Universität Bremen
Zentrum für Technomathematik
FB 03: AG Optimierung & Optimale Steuerung
Postfach 33 04 40
28334 Bremen
bueskens@math.uni-bremen.de

Fritz Colonius

Universität Augsburg
Institut für Mathematik
Universitätsstraße 14
86135 Augsburg
fritz.colonius@math.uni-augsburg.de

Eva Crück

Laboratoire d'Applications des
Systèmes Tychastiques Régulés
14, rue Domat
75005 Paris
France
eva.cruck@polytechnique.edu

Moritz Diehl

Katholieke Universiteit Leuven
Department of Electrical Engineering
Optimization in Engineering Center (OPTEC)
Kasteelpark Arenberg 10
3001 Leuven-Heverlee
Belgium
moritz.diehl@esat.kuleuven.be

Tzanko D. Donchev

University of Architecture and Civil Engineering
Department of Mathematics
1 "Hr. Smirnenski" Str.
1046 Sofia
Bulgaria
tzankodd@gmail.com

Assen L. Dontchev

Program Director
Analysis Program
Division of Mathematical Sciences
National Science Foundation
4201 Wilson Blvd., Room 1025
Arlington, VA 22230, U.S.A.
adontche@nsf.gov

Elza Farkhi

Tel-Aviv University
Sackler Faculty of Exact Sciences
School of Mathematical Sciences
Department of Applied Mathematics
Ramat Aviv, Tel-Aviv 69978,
Israel
elza@post.tau.ac.il

Tatiana F. Filippova

Russian Academy of Sciences
Institute of Mathematics and Mechanics
Department of Optimal Control
16, S.Kovalevskaja str.
Ekaterinburg 620219
Russia
ftf@imm.uran.ru

Matthias Gerdts

University of Birmingham
School of Mathematics
Watson Building
Edgbaston
Birmingham B15 2TT
Great Britain
gerdtsm@maths.bham.ac.uk

Lars Grüne

Universität Bayreuth
Mathematisches Institut
Professur für Angewandte Mathematik
95440 Bayreuth
`lars.gruene@uni-bayreuth.de`

Willi Jäger

Universität Heidelberg
Institut für Angewandte Mathematik und IWR
Im Neuenheimer Feld 294
69120 Heidelberg
`jaeger@iwr.uni-heidelberg.de`

Peter E. Kloeden

Johann Wolfgang Goethe–Universität
Fachbereich Mathematik
AG Numerik, Dynamik und Optimierung
Postfach 11 19 32
60054 Frankfurt am Main
`kloeden@math.uni-frankfurt.de`

Frank Lempio

Universität Bayreuth
Mathematisches Institut
Lehrstuhl für Angewandte Mathematik
95440 Bayreuth
`frank.lempio@uni-bayreuth.de`

Thomas Lorenz

Universität Heidelberg
Interdiszipl. Zentrum für Wissenschaftl. Rechnen
Im Neuenheimer Feld 294
69120 Heidelberg
`thomas.lorenz@iwr.uni-heidelberg.de`

Helmut Maurer

Westfälische Wilhelms-Universität Münster
Fachbereich Mathematik
Institut für Numerische
und Angewandte Mathematik
Einsteinstr. 62
48149 Münster
`maurer@math.uni-muenster.de`

José Alberto Murillo Hernández

Universidad Politécnica de Cartagena
Departamento de Matemática Aplicada
y Estadística
Campus Muralla del Mar
C/ Dr Fleming, s/n
30202 Cartagena
Spain
`alberto.murillo@upct.es`

Jürgen Pannek

Universität Bayreuth
Mathematisches Institut
Lehrstuhl für Angewandte Mathematik
95440 Bayreuth
`juergen.pannek@uni-bayreuth.de`

Janosch Rieger

Universität Bielefeld
Fakultät für Mathematik
Postfach 100131
33501 Bielefeld
`jrieger@mathematik.uni-bielefeld.de`

Jan-J. Rückmann

University of Birmingham
School of Mathematics
Watson Building
Edgbaston
Birmingham B15 2TT
Great Britain
`ruckmanj@maths.bham.ac.uk`

Patrick Saint-Pierre

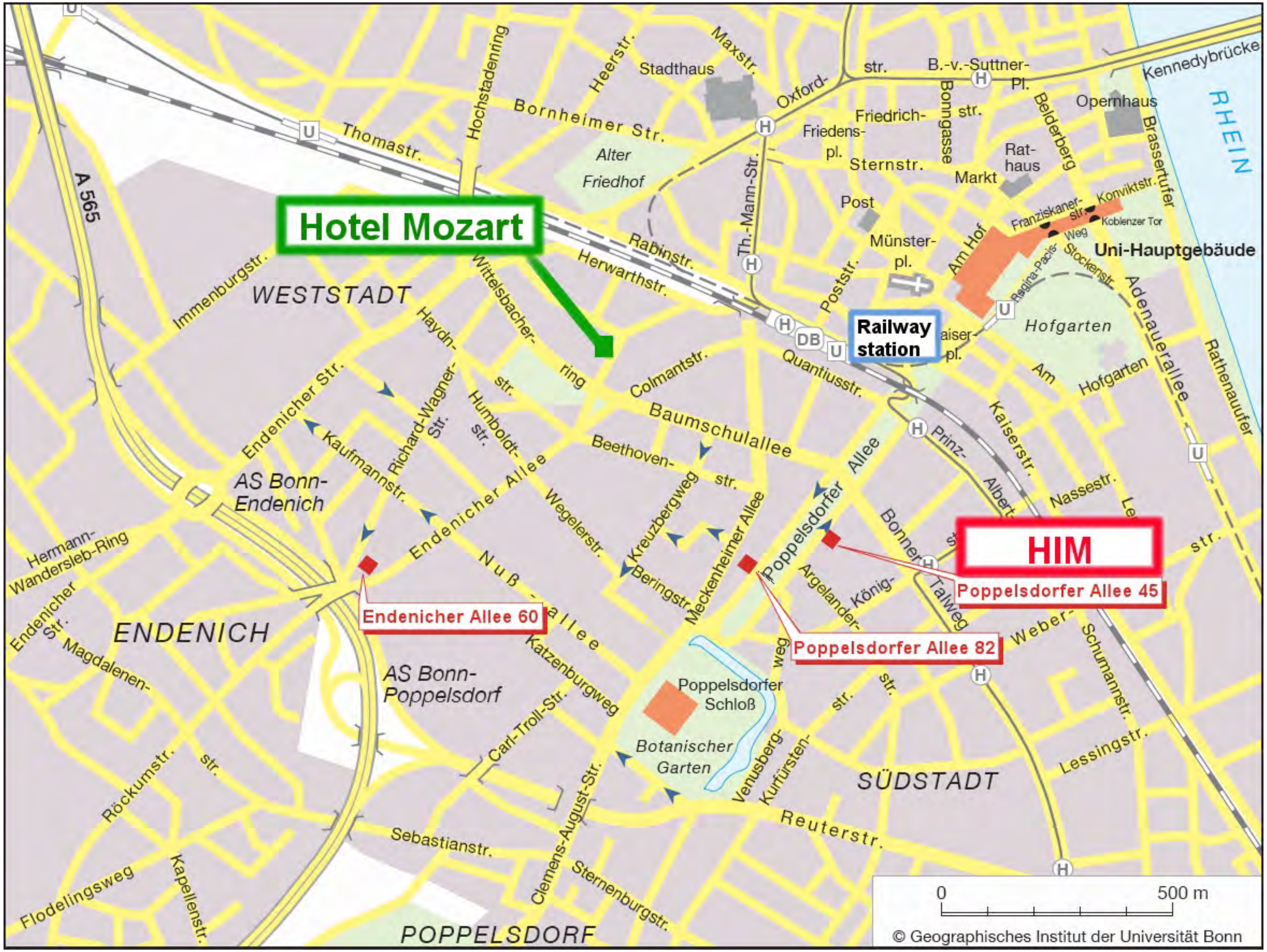
Université Paris-Dauphine
SDFI–VJC (EA 3393)
Place du Maréchal de lattre de Tassigny
75775 Paris Cedex 16
France
`patrick.saint.pierre@gmail.com`

Vladimir Veliov

Technische Universität Wien
Institut für Wirtschaftsmathematik
Operations Research & Nichtlin. Dynam. Systeme
Argentinierstr. 8/119
1040 Wien
Österreich
vveliov@eos.tuwien.ac.at

Karl Worthmann

Universität Bayreuth
Mathematisches Institut
Lehrstuhl für Angewandte Mathematik
95440 Bayreuth
karl.worthmann@uni-bayreuth.de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Programme of Workshop on

Set-valued Numerical Analysis and Robust Optimal Control

	Thursday, March 27
9.00 – 9.15	*Welcome*
9.15 – 10.00	**A. Dontchev** Implicit function theorems: Old and new
10.15 – 11.00	**Colonius** Some numerical problems for stability of deterministic and stochastic systems
11.15 – 11.45	*Break*
11.45 – 12.30	**Kloeden** The numerical solutions of stochastic differential equations with a discontinuous drift coefficient
12.45 – 14.15	*Lunch*
14.15 – 15.00	**Saint-Pierre** How to control complex dynamical system: A generation of viability algorithm
15.15 – 15.35	**T. Donchev** Invariance and some other properties of impulsive systems
15.40 – 16.15	*Break*
16.15 – 17.00	**Filippova** State estimation approaches for control systems under uncertainty
17.15 – 17.40	**Gerdts** Approximation of reachable sets using optimal control algorithms
18.00	*HIM reception*

	Friday, March 28
9.15 – 9.45	**Maurer** Retarded optimal control problems with an application in biomedicine
10.00 – 10.45	**Maurer** Sensitivity analysis and real-time control of bang-bang and singular control problems
11.00 – 11.30	*Break*
11.30 – 12.15	**Grüne** A set-oriented min-max approach to robust optimal control under coarse discretization and quantization
12.30 – 14.00	*Lunch*
14.15 – 15.00	**Diehl** Practical approaches to robust optimal control and real-world experiments in engineering applications
15.15 – 16.00	**Rückmann** On stable feasible sets in generalized semi-infinite programming
16.15 – 16.45	*Break*
16.45 – 17.05	**Murillo Hernández** The heat equation in noncylindrical domains governed by morphological equations
17.10 – 17.35	**Lorenz** Control problems for nonlocal set evolutions with state constraints
19.00	*Joint dinner*

	Saturday, March 29
9.00 – 9.45	**Lempio** Discrete approximation of differential inclusions
10.00 – 10.45	**Veliov** Some problems and new results concerning discrete approximations of control systems
11.00 – 11.30	*Break*
11.30 – 12.15	**Farkhi** Discrete approximations and relaxation of one-sided Lipschitz differential inclusions
12.30 – 14.00	*Lunch*
14.15 – 14.35	**Rieger** Shadowing in set-valued dynamical systems
14.40 – 15.00	**Baier** Set-valued Hermite interpolation, a case study of directed sets
15.05 – 15.40	*Break*
15.40	**Closing discussion** ... with open end ...